

1. Let $A = \begin{bmatrix} 5 & -7 & -4 \\ 6 & -8 & -7 \\ 4 & -4 & -9 \end{bmatrix}$. (a) Find the $\text{adj}A$. (b) Find A^{-1} by using the $\text{adj}A$.

Answer: $A^{-1} = \frac{1}{6} \begin{bmatrix} 44 & -47 & 17 \\ 26 & -29 & 11 \\ 8 & -8 & 2 \end{bmatrix}$.

2. Let S be the parallelogram determined by the vectors $u = \begin{bmatrix} 3 \\ 7 \end{bmatrix}$ and $v = \begin{bmatrix} -2 \\ 4 \end{bmatrix}$.

Let T be the linear transformation defined by $T(x) = Ax$, where $A = \begin{bmatrix} 2 & -1 \\ 1 & 3 \end{bmatrix}$.

Find the area of $T(S)$. (**Ans:** Area= 182 unit²).

3. Let S be the parallelepiped determined by the vectors $u = \begin{bmatrix} 3 \\ 1 \\ 2 \end{bmatrix}$, $v = \begin{bmatrix} 4 \\ 5 \\ 1 \end{bmatrix}$, $w = \begin{bmatrix} 1 \\ 2 \\ 4 \end{bmatrix}$.

Let T be the linear transformation defined by $T(x) = Ax$, where $A = \begin{bmatrix} -2 & 1 & 4 \\ 3 & 5 & 7 \\ 6 & 0 & -1 \end{bmatrix}$.

Find the volume of $T(S)$. (**Ans:** Volume= 2925 unit³).

4. For each of the following matrices

$$A = \begin{bmatrix} 1 & 4 & 0 \\ 0 & 5 & 0 \\ 1 & 0 & 1 \end{bmatrix}, \quad B = \begin{bmatrix} 3 & 0 & -2 \\ -7 & 0 & 4 \\ 4 & 0 & -3 \end{bmatrix}, \quad C = \begin{bmatrix} 2 & 0 & 0 & 0 \\ 0 & 2 & 0 & 0 \\ 1 & 4 & -1 & 0 \\ -1 & -2 & 0 & -1 \end{bmatrix}:$$

- Find the eigenvalues and the corresponding eigenvectors.
- For each eigenvalue, write down the corresponding eigenspace.
- Diagonalize the matrix, if possible.

5. Decide whether each of the following statements is TRUE or FALSE:

- If all the eigenvalues of a matrix A are 0, then A is the zero matrix.
- If 1 is the only eigenvalue of a 2×2 matrix A , then $A = I$.
- If $\lambda = 0$ is an eigenvalue of a matrix A , then $\det A = 0$.
- If a matrix A is invertible, then $\lambda = 0$ is not an eigenvalue of A .
- If the characteristic polynomial of a 3×3 matrix A is $p(\lambda) = (\lambda - 3)^2(\lambda + 5)$, then $\dim E_3 = 2$.
- If $\lambda = 7$ is an eigenvalue of a matrix A , then $\det(A - 7I) = 0$.

6. A is a 2×2 matrix with eigenvectors $v_1 = \begin{bmatrix} 1 \\ -2 \end{bmatrix}$ and $v_2 = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$ corresponding to eigenvalues

$\lambda_1 = -1$ and $\lambda_2 = 2$, respectively.

(a) If $x = \begin{bmatrix} 5 \\ -1 \end{bmatrix}$, find Ax without finding A .

(b) Find the matrix A .

(c) Find the matrix A^8 .